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(STANDARDS)

NATIONAL ROADS BOARD
HIGHWAY STANDARDS S/2

HORIZONTAL AND VERTICAL CURVES FOR HIGHWAYS

CONTENTS

Note. Transition Curve Data Tables have been issued to be used in conjunction with these Standards.

PART I. HORIZONTAL CURVES

1. INTRODUCTION AND THEORY:

(a) Transition Curves:

Transition curves are curves of varying radii, which are used in conjunction with superelevation, to minimize the effect of sudden changes in the centrifugal force acting on a vehicle and passengers. When a body passes from a straight to a curved path, the forces acting on it are altered. If the path of a body changes instantaneously from a straight line to a circular curve of fixed radius, the centrifugal force will be applied instantaneously and the body will experience shock. If, however, the radius is reduced gradually from infinity to the minimum radius, the magnitude of the centrifugal force acting on a vehicle and passengers will be increased gradually, and very little shock will be experienced.

Two types of transition curves in general use are the cubic spiral and the lemniscate. For highway transition curves, where deflection angles are large, the lemniscate curve is preferred. The radius of curvature does not decrease as rapidly in the lemniscate as it does in the spiral, and this makes for greater safety. In addition, the lemniscate has a valuable property from the setting out point of view, in that the exterior deflection angle between the end tangents is always exactly three times the polar deflection angle "a".

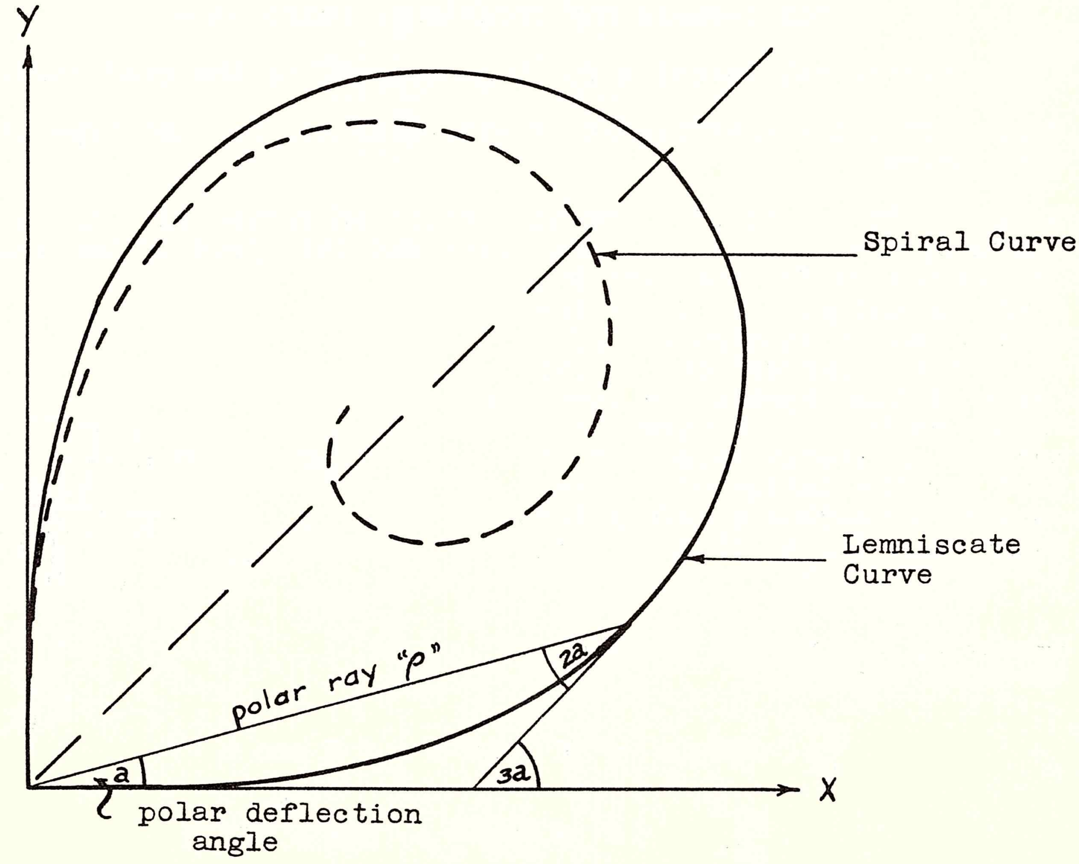


Fig.1
Illustration Comparing the Spiral and Lemniscate Curves.

The Basic equation of the lemniscate curve is:-

$$\rho = C\sqrt{\sin 2a}$$

where "ρ" is the length of the polar ray, "C" is a constant, and "a" is the polar deflection angle.

Engineers interested in studying the theory and the geometry of transition curves are recommended to read Professor Royal Dawson's book, "Elements of Curve Design for Road, Railway and Racing Track on Natural Transition Principles" or Clarke's "Plane and Geodetic Surveying", Volume I.

(b) Superelevation:

The total superelevation on a horizontal curve is the height the outer edge of the road is raised above the inner edge. For convenience, superelevation is usually referred to as a "rate of superelevation", expressed in inches per foot of roadway width. The following is a brief description of the theory used in compiling the superelevation and widening diagram :-

The centrifugal force which acts on a vehicle travelling around a curve tends to make the vehicle move sideways and towards the outer edge of the road. On a superelevated curve this centrifugal force is balanced partly by the superelevation and partly by the frictional resistance of the tyres on the road surface.

The formula for centrifugal force is:-

$$P = \frac{WV^2}{gR}$$

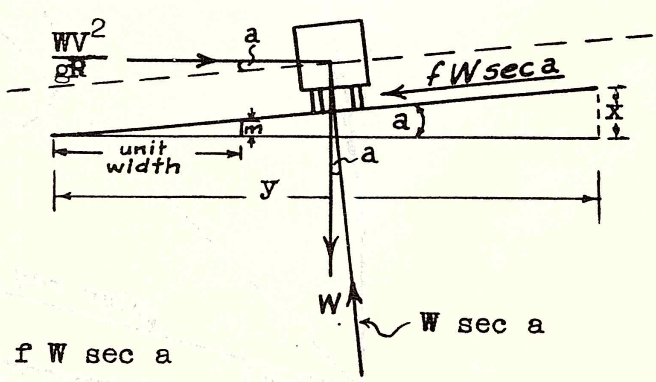
Where P = centrifugal force, W = weight of vehicle, g = acceleration due to gravity in ft/sec^2 , V = velocity in ft/sec, and R is the radius in feet.

The formula for frictional force is:-

(Frictional force) = f. (force normal to the road surface).

Where "f" = the coefficient of side friction for the type of road surface considered.

Note:- The coefficient of side friction varies with the type of road surface, the condition of the tyres and the speed of the vehicle. Limiting values of "f" for dry rough surfaces and good tyres at low speeds can be as high as 0.7, while those for wet smooth surfaces at high speeds are sometimes as low as 0.2. However, the maximum allowable values of "f" used for the design of highway curves, including a safety factor, range from 0.2 at 15 m.p.h. to 0.1 at 80 m.p.h.



Resolving the forces parallel to the road surface:-

$$\frac{WV^2}{gR} \cos a = W \sin a + f W \sec a$$

$$\frac{WV^2}{gR} = W\frac{\sin a}{\cos a} + \frac{fW}{\cos a . \cos a}$$

$$\frac{WV^2}{gR} = W \tan a + fW \quad \begin{pmatrix}\text{Where a is small} \\ \cos a = 1 \text{ approx.}\end{pmatrix}$$

$$\therefore \frac{V^2}{gR} = \tan a + f$$

$$\text{but } \tan a = \frac{x}{y} = \frac{m}{1} = m$$

Where "m" = the superelevation per unit width. (ie., in feet per foot).

$$\therefore \text{ Centrifugal Ratio} \left(\frac{P}{W}\right) = \frac{V^2}{gR} = m + f.$$

This equation is used to determine the "absolute" minimum radius for a particular speed value, and includes the maximum superelevation ($1\frac{1}{2}$ inches per foot) and the maximum allowable coefficient of side friction.

$$\left(\begin{array}{l}\text{Where S is m.p.h.}\\ \text{and R is in feet}\end{array}\right) \qquad m + f = \frac{S^2}{15R}$$

$$\therefore R = \frac{S^2}{15\,(m + f)}$$

On curves where less than maximum superelevation is employed, the coefficient of side friction is reduced proportionately. At any radius, the value of superelevation "m" and the value for the coefficient of side friction "f" are such, that $\frac{m}{(f+m)}$ (i.e., the ratio of superelevation to centrifugal ratio) is a constant for a particular speed value. E.g., for any point on a 40 m.p.h. curve :-

$$\frac{m}{(m+f)} = \frac{.125}{.125 + .150} = \frac{.125}{.275} = \frac{5}{11}$$

$$\therefore \text{ Superelevation per unit width at any point} \qquad m' = \frac{V^2}{gR} \cdot \frac{m}{(m+f)}$$

$$\left(\begin{array}{l}\text{Superelevation in inches}\\ \text{per foot of roadway}\\ \text{width}\end{array}\right) = \frac{12\,S^2}{15\,R} \cdot \frac{.125}{(f + .125)}$$

(c) <u>Tables and Charts:</u>

Transition curve data tables, which include tables A,B and C, together with Appendices I and II, have been prepared to be used in conjunction with these standards for both design and field calculations.

Table A is for use in the design of curves with central circular arcs, and provides data for setting out <u>all</u> transitions. Table B gives design standards and constants, but will generally be used solely for the purpose of selecting a suitable unit chord with which to set out a curve. Table C is for use in the design of curves transitional throughout their length. These tables are obtainable in either links or feet.

Appendix I is the base from which table A has been computed and Appendix II is the base from which table C has been computed. All the values in these tables are in terms of unit chord lengths. Although nearly all transitions can be designed with the use of tables A,B and C; ther are, however, a few situations which cannot be satisfied without varying the unit chord length. In these cases, it is necessary to use the original tables in the Appendices, and to calculate the values by multiplying by the chord length chosen.

A superelevation and widening diagram is included with the tables.

The use of the various tables and charts is explained in the following paragraphs.

2. DESIGN OFFICE PROCEDURE.

(a) Pre-requisite Data and Choice of a Suitable Curve.

Non Transitional Circular Curves. Pure circular curves without transitions may be used only with a radius of 40 chains or more, or where the deviation angle is less than 7 degrees.

Transitional Curves. Horizontal curves shall generally be constructed as circular arcs joined to their tangents by transitions, but where physical conditions render the insertion of central circular arcs impossible or undesirable, the curves shall be transitional throughout their length. In hilly country, the limiting factor will often be overlapping tangents of adjacent curves, and it will be found in many cases that purely transitional curves are necessary.

Small Deviation Angles. Where deviation angles are small, horizontal curves should be sufficiently long to avoid the appearance of a kink. Where possible, curves should be at least 6 chains long for a deviation angle of 5 degrees, and the minimum length should be increased by at least 1 chain for each 1 degree decrease in the deviation angle. To achieve this, large radius circular curves or transitions of a higher speed value may be used.

Maximum and Minimum Superelevation. Whereas a superelevation of less than 1 inch per foot of width is considered desirable for all classes of traffic, the topography of the country or other limiting features sometimes necessitate the use of curves having the maximum permissible superelevation of $1\frac{1}{2}$ inches per foot in order to obtain the required speed value. However, every effort should be made to keep the maximum superelevation below $1\frac{1}{4}$ inches per foot. Where horizontal curves occur on a steep gradient or where vehicles such a double deck sheep trucks frequently travel on a highway, the maximum superelevation should be restricted.

Suggested Maximums:-

Where double deck trucks are in use	-	1" per foot.
Gradients between 1 in 20 and 1 in 10	-	$1\frac{1}{4}$" " "
Gradients steeper than 1 in 10	-	1" " "

The minimum rate of superelevation on any central circular arc or centre point should equal the rate of normal camber (i.e., $\frac{3}{8}$" per foot) continued for the full roadway width. This value will provide adequate drainage and improve the appearance of a curve where the calculated superelevation is less than $\frac{3}{8}$" per foot. It is advisable to include a central circular arc at least 2 chains long in all curves where such low values of superelevation are employed.

Pre-requisite Data. The deviation angle must be determined and the speed value fixed, before any curve can be designed.

If the deviation angle is greater than the maximum listed in table C for the particular speed value, a central circular arc joined to its tangents by transitions must be used. The use of a fully transitional curve under these circumstances would result in the maximum permissible superelevation being exceeded.

If the deviation angle is smaller than the maximum listed in Table C for the particular speed value, a curve transitional throughout may be used. However, where there are no site limitations, a curve having a central circular arc joined to its tangents by transitions, is preferred, even if the deviation angle is less than the maximum deviation angle listed in table C.

(b) Use of Tables and Calculations.

Curves with Central Circular Arcs Joined to their Tangents by Transitions. Having ascertained the deviation angle and decided upon the speed value, careful consideration must now be given to the most important item in this type of curve design, the radius of the central circular arc.

Features, such as steep cliffs, rivers and buildings often limit the degree of curvature that can be employed. Where physical limitations do not exist, the superelevation diagram should be used to determine the most suitable radius for the central circular arc. With due consideration to the cost involved, designers should aim at achieving the required speed value with a minimum amount of superelevation.

From the superelevation diagram, find the radius of the curve that will give the required speed value with a superelevation of $\frac{1}{2}$ inch per foot of width. If the radius so obtained relates to a curve unsuitable to the site, try curves of decreasing radii until a suitable curve has been found. These preliminary trials can be carried out on paper using circular railway curves.

Having now determined by trial and error the radius of the curve best suited to the site, select from the appropriate column in table A, the minimum radius R, nearest in numerical value to the one just determined. This radius now becomesthe radius of the central circular arc. The superelevation is adjusted accordingly.

From table A ascertain the following:-

The length of transition L, i.e., the distance measured along the curve from the tangent point (T.P.) with the straight, to the common tangent point (C.T.P.) with the central circular arc.

The deflection angle a, i.e., the angle between the main tangent line and the polar ray to the C.T.P. from the T.P.

The shift s, i.e., the distance the circular arc has to be moved from the main tangent to make provision for the transition.

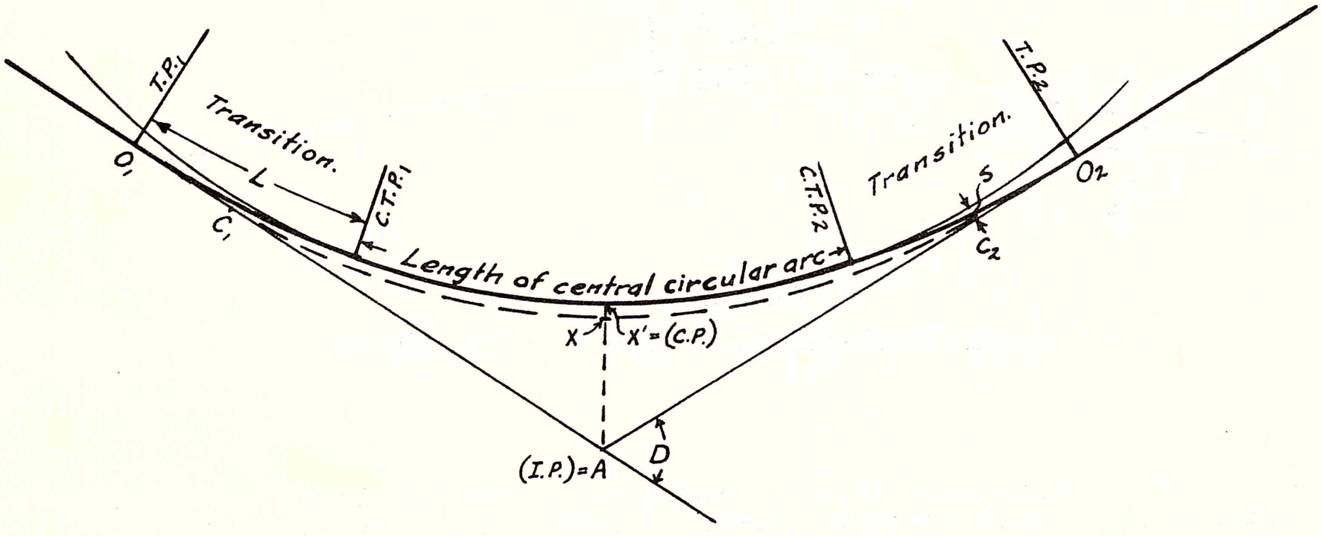
The correction t.

The tangent length AO, i.e., the distance between the intersection point of the tangents (I.P.) and the tangent point (T.P.). This is calculated from the following formula:-

$$AO = AC + CO$$

$$\text{where } AC = (R + s) \tan \frac{D}{2}$$

$$\text{and } CO = \tfrac{1}{2} L - t$$



<u>Fig. 2.</u>

<u>Curve with central Circular Arc joined to its Tangents by Transitions.</u>

The apex distance AX', i.e., the distance from the intersection point (I.P.) to the mid point of the central circular arc,(C.P.) is calculated from the following formula:-

$$AX' = AX + XX'$$

$$\text{where } AX = (R + s)\left(\sec \frac{D}{2} - 1\right)$$

$$\text{and } XX' = \text{the shift } s$$

The length of the central circular arc, i.e., the distance around the curve from C.T.P.$_1$ to C.T.P.$_2$. This is calculated from the following formula:-

$$\text{Length of circular arc} = (D - 6a).\ R$$

Where the angle (D - 6a) is in radians, and R is the radius of the central circular arc.

The total length of the curve, i.e., the distance from T.P.$_1$ to T.P.$_2$.

Total length = 2L + length of circular arc.

Note: Co-ordinates for plotting purposes can be obtained from the required speed value section of Table A.

Curves Transitional throughout their Length. From table C, for the given deflection angle and speed value determine by interpolation the following:-

The minimum radius R.

The curve length L, i.e., the distance measured along the curve between the tangent point (T.P.) and the mid point of the transitions (C.P.).

The tangent length AO, i.e., the distance between the intersection point of the tangents (I.P.) and the tangent point (T.P.).

The apex distance AX, i.e., the distance from the intersection point (I.P.) to the point of minimum radius at the mid point of the curve (C.P.). The mid point of the curve lies on the bisector of the angle of intersection.

The total length of the curve = 2L.

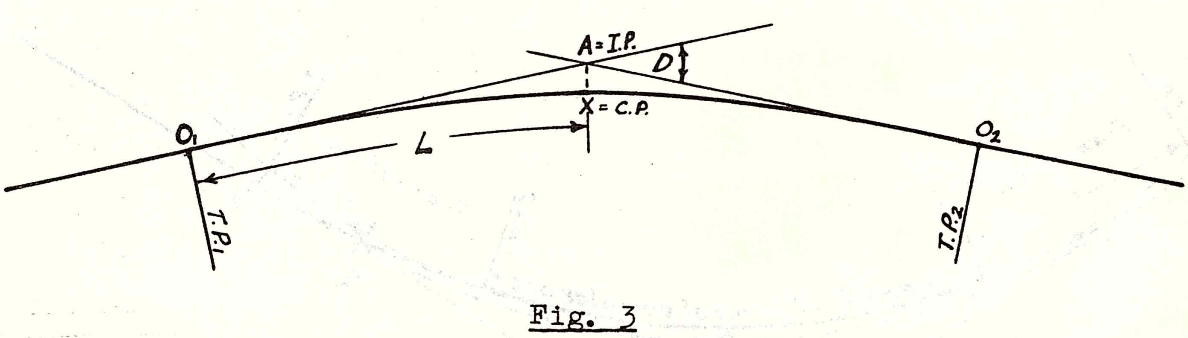


Fig. 3

Curve Transitional throughout its length.

Note:- For preliminary trials, transition curve templates may be used to determine the most suitable curve. Transition curve templates for speed values varying by 5 m.p.h., and at scales of 1 chain to 1 inch and 3 chains to 1 inch, can be obtained from the Head Office of the Ministry of Works. However, any transition can be quickly sketched in by plotting the tangent length obtained from table C and plotting co-ordinates from table A.

Superelevation. The rate of superelevation at the point of minimum radius is determined from the superelevation and widening diagram, and intermediate values are calculated by direct proportion to their distances from the tangent point (T.P.).

$$\text{i.e.}\ \begin{pmatrix}\text{Rate of Super. at}\\ \text{any point}\end{pmatrix} = \begin{pmatrix}\text{Rate of Super. at}\\ \text{point of min. rad.}\end{pmatrix} \cdot \frac{\begin{pmatrix}\text{distance of the point}\\ \text{from the T.P.}\end{pmatrix}}{\text{Length of transition (L).}}$$

The maximum rate of superelevation can be determined more accurately if required from the following formula:-

$$\begin{pmatrix}\text{Superelevation in inches}\\ \text{per foot of width}\end{pmatrix} = \frac{1.212S^2}{R} \cdot \begin{pmatrix}\text{Proportion of superelevation to}\\ \text{Centrifugal ratio from Table B}\end{pmatrix}$$

Where S = speed value in M.P.H.

and R = minimum radius in links.

The distance "x" can be calculated as follows:-

$$x = (\text{length of transition}) \cdot \frac{(\text{rate of camber})}{\begin{pmatrix}\text{rate of maximum}\\ \text{superelevation}\end{pmatrix}}$$

Application of Superelevation. (Reference Fig. 4). Assuming normal camber at a point M, a distance "x" back from the T.P., the outer edge of the road is raised uniformly about the centre line until the tangent point is reached, where the outer half of the road is level, and the inner half of the road is at normal camber. The outer edge of the road is raised about the centre line still further until a point N is reached, a distance "x" forward of the tangent point, where the rate of superelevation is constant across the roadway width, and equal to the rate of normal camber. The whole roadway is now tilted uniformly about the inner edge of the formation, up to the point of minimum radius at the C.P. or the C.T.P., where the rate of superelevation is a maximum.

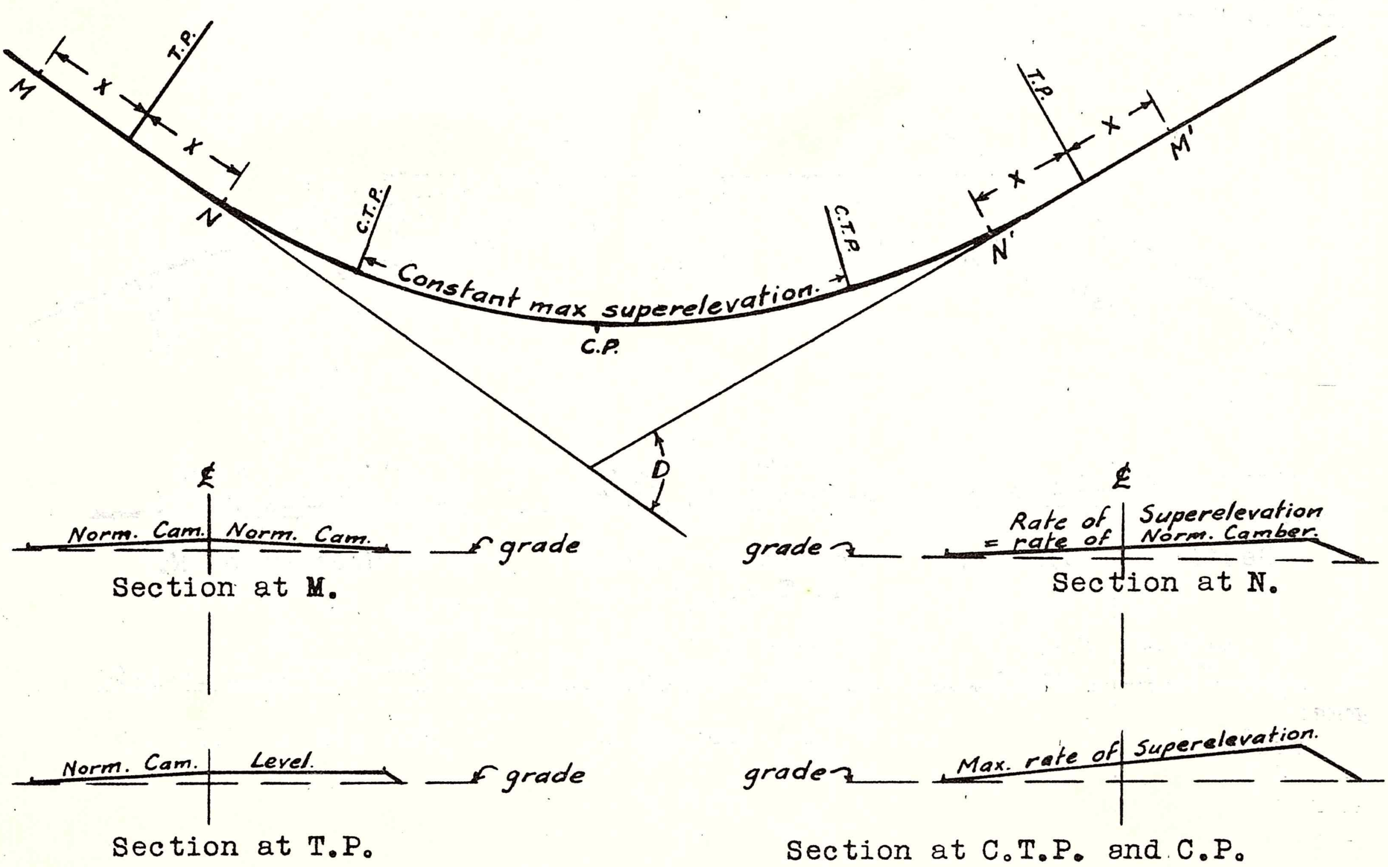
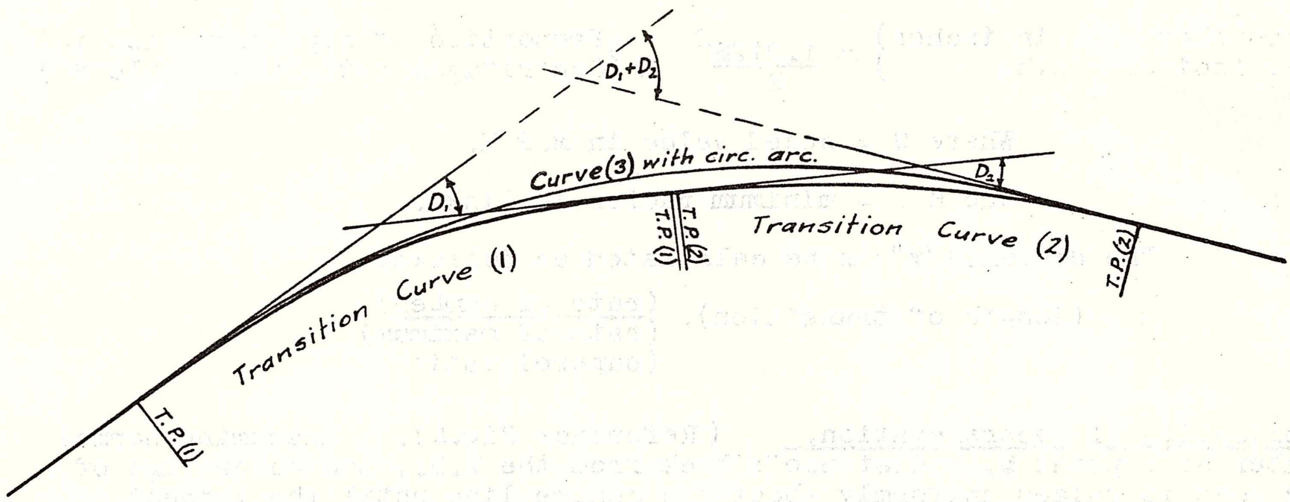


Fig. 4
Application of Superelevation.

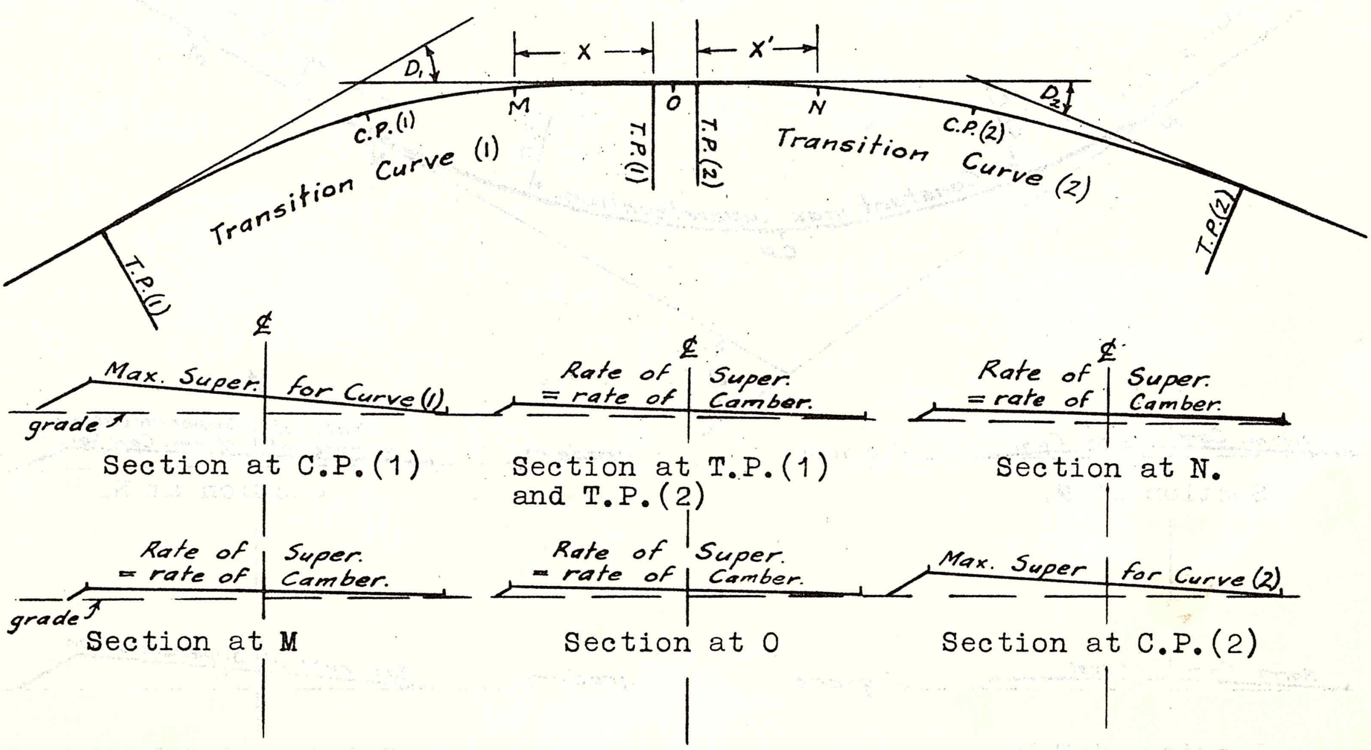
If the curve has a central circular arc the maximum superelevation is constant throughout the length of the central circular arc, and then begins to decrease from the C.T.P. If the curve is transitional throughout, the rate of superelevation begins to decrease immediately after the point of minimum radius has been reached.

<u>Elimination of "Broken Back" Alignment.</u> Where the tangent points of two curves are very close together, and the rate of superelevation is applied in the usual manner, a "broken back" effect is produced. This can be avoided by replacing the curves by a single curve as shown in Fig.5.



<u>Fig. 5.</u>
<u>Replacement of two adjacent transition curves by one curve with a central circular arc transitioned at each end.</u>

Where the curves are in the same directions with tangent points very close together, and it is impossible to replace them by a single curve, the rate of superelevation in between should be maintained constant from section M to section N as shown in Fig.6. (x and x' are distances from the T.P's where the rate of superelevation = rate of normal camber).



<u>Fig. 6</u>

<u>Adjustment of superelevation for two curves in the same direction with Tangent points close together.</u>

Where the curves are in opposite directions, with tangent points very close together, the rate of superelevation in between should be decreased gradually from a maximum at the centre of one curve to a level section at the tangent and then remain level to the next tangent, increasing gradually to a maximum at the centre of the other curve. (x and x' are distances from the T.P's where the rate of superelevation = rate of normal camber). Care must be taken to grade the length between adjacent tangent points, in which there are level cross sections to give adequate longitudinal drainage.

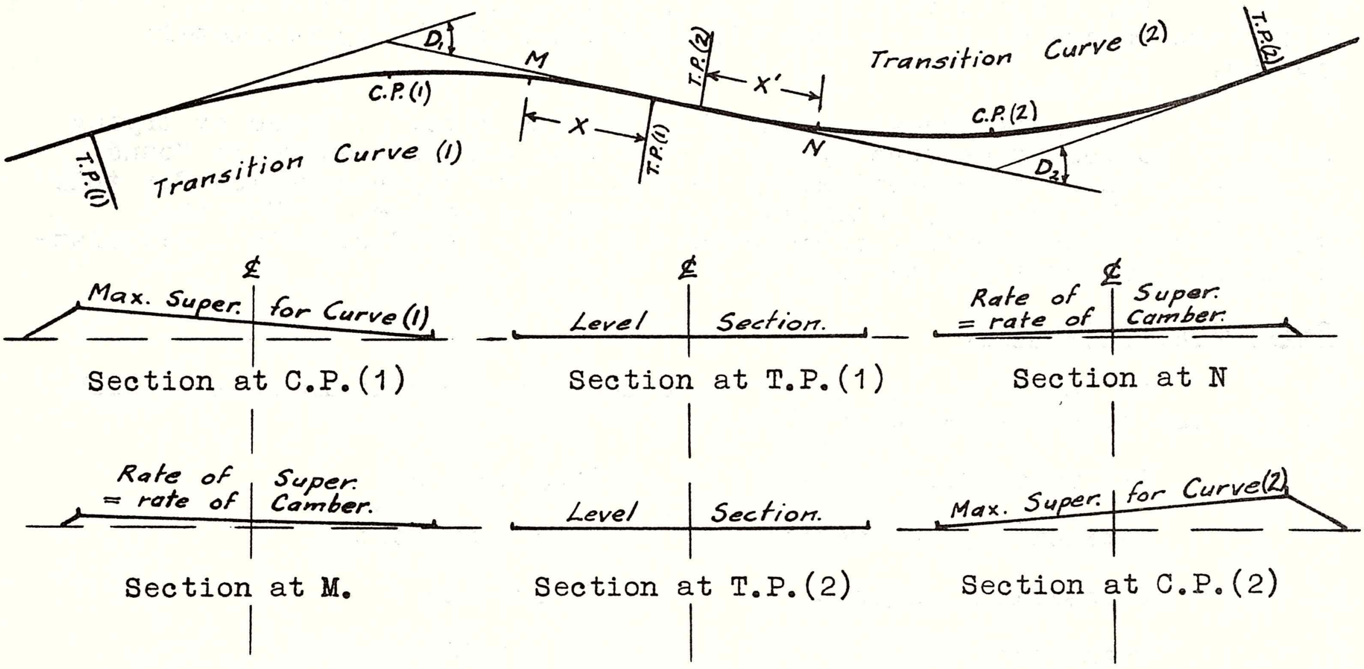


Fig. 7

Adjustment of Superelevation for two curves in opposite directions with Tangent points close together.

__Widening.__ Determine the total widening at the point of minimum radius from the superelevation and widening diagram, and calculate intermediate values by direct proportion to their distances from the tangent point (T.P.).

$$\text{i.e.}\ \begin{pmatrix}\text{Widening at}\\ \text{any point}\end{pmatrix} = \begin{pmatrix}\text{Widening at point}\\ \text{of min. rad.}\end{pmatrix} \cdot \frac{\begin{pmatrix}\text{Distance of the point}\\ \text{from the T.P.}\end{pmatrix}}{\text{Length of Transition (L)}}$$

For a curve having a central circular arc, widening shall be zero at the tangent point, increase uniformly to a maximum at the common tangent point, remain a maximum to the second common tangent point, and then decrease uniformly to zero at the second tangent point. For a curve transitional throughout, widening shall be zero at the tangent point, increase uniformly to a maximum at the centre point of the curve, and then decrease uniformly to zero at the second tangent point.

Half the widening obtained from the superelevation and widening diagram shall be applied to each side of the road.

Where two curves have their tangent points very close together, the full amount of widening shall be applied for the distance between them.

3. WORKED EXAMPLES.

Example I.

Curve with Central Circular Arc Transitioned at each end.

Design a curve for a speed value of 50 m.p.h. and a deflection angle of $65^\circ\ 04'\ 30''$.

Referring to the Superelevation Diagram.

For a speed value of 50 m.p.h. and superelevation of 0.5" per foot, the radius of the central circular arc would be approximately 30 chs.

This curve proves unsuitable for the site. Hence by trying circular curves of decreasing radii, the most suitable curve is found to have a radius of approximately 19 chs. The superelevation for this curve is 0.8" per foot, which is considered reasonable. Now that the radius of the central circular arc has been decided, detail calculations for the curve can be made.

Referring to Table A.

In the 50 m.p.h. speed value section a radius is chosen which is nearest numerically to 19 chs.

Radius of Circular Arc R	=	1845.43 lks.
Maximum superelevation	=	0.83" per foot
Transition length L	=	321.97 lks.
Deflection angle "a"	=	$1^\circ\ 40'\ 00''$
Co-ordinates at C.T.P.	=	321.74 and 9.36 (measured from the T.P.)
Shift s	=	2.34 lks.
Correction t	=	0.08 lks.

Tangent Length AO.

AO $= AC + CO$

where AC $= (R + s) \tan \frac{D}{2}$

$= (1847.77) \tan 32^\circ\ 32'\ 15''$

log AC $= \log 1847.77 + \log \tan 32^\circ 32' 15''$

$= 3.2666480 + 9.8048143$

$= 13.0714623 \equiv 3.0714623$

$\therefore$ AC $= 1178.86$

Now CO $= \frac{1}{2} L - t$

$= 160.98 - 0.08 = 160.90$

$\therefore$ <u>AO</u> $= 1178.86 + 160.90 =$ <u>1339.76 lks.</u>

Distance from the intersection of the straights to the centre point of the curve (AX').

AX'	=	AX + XX'
Where AX	=	$(R + s) \left(\operatorname{Sec} \frac{D}{2} - 1\right)$
	=	$1847.77 (\sec 32^\circ\ 32'\ 15'' - 1)$
	=	1847.77 (0.186184)
Log AX	=	log 1847.77 + log 0.186184
	=	$3.2666480 + \bar{1}.2699424$
	=	2.5365904
∴ AX	=	344.03
Now XX'	=	s = 2.34
∴ AX'	=	344.03 + 2.34 = 346.37 lks.

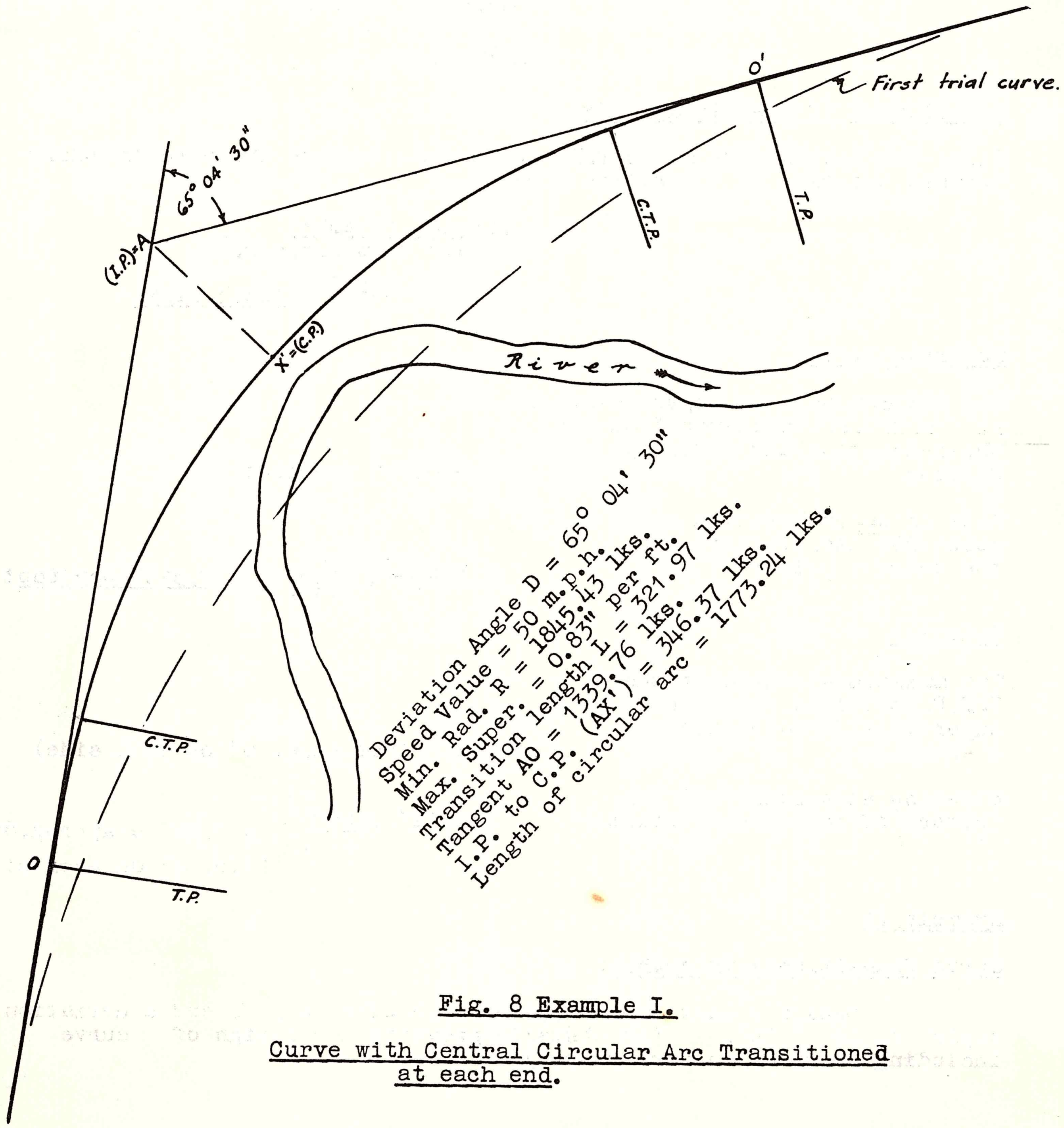


Fig. 8 Example I.

Curve with Central Circular Arc Transitioned at each end.

Length of Circular Arc.

Where the angle (D-6a) is measured in radians and R is the radius of the central circular arc in links.

$$\left(\begin{matrix}\text{Length of circular}\\ \text{arc in links}\end{matrix}\right) = (D - 6a).R$$

$$D - 6a = 65^\circ\ 04'30'' - 10^\circ\ 00'\ 00'' = 55^\circ\ 04'\ 30''$$

$$= 0.9612400 \text{ Radians.}$$

$$\log \left(\begin{matrix}\text{Length of}\\ \text{circular arc}\end{matrix}\right) = \log 0.96124 + \log 1845.43$$

$$= \bar{1}.9828318 + 3.2660976$$

$$= 3.2489294$$

$$\therefore \left(\begin{matrix}\text{Length of}\\ \text{circular arc}\end{matrix}\right) = \underline{1773.90 \text{ links}}.$$

Total length of Curve.

$$\text{Total length} = 2L + \text{length of circular arc}$$

$$= 643.94 + 1773.24$$

$$= \underline{2417.18 \text{ lks}}.$$

Calculation of the Distance "x".

(i.e. the distance from the T.P. to the point where the rate of camber equals the rate of superelevation)

$$x = L.\frac{(\text{rate of camber})}{(\text{rate of max. super.})}$$

$$= 321.97 \cdot \frac{0.375}{0.83} = \underline{145.3 \text{ links}}$$

Superelevation.

The maximum rate of superelevation at the C.T.P., obtained from the superelevation and widening diagram = 0.83" per foot.

Rate of superelevation at a point 200 lks. forward of the tangent point $= \frac{0.83}{1} \cdot \frac{200}{321.97} = \underline{0.515'' \text{ per foot}}$

Widening.

The maximum widening at the C.T.P. obtained from the superelevation and widening diagram = 1 foot (i.e. 6" on each side)

Widening at a point 200 lks. forward of the tangent point $= 12'' \cdot \frac{200}{321.97} = 7.46'' =$ approx. 8" (i.e. 4" on each side)

Example II

Curve Transitional Throughout.

Design a curve for a speed value of 45 m.p.h. and a deviation angle of $21^\circ\ 4'\ 30''$. (An obstacle prevents the design of a curve including a central circular arc.)

Referring to Table C.

In the 45 m.p.h. section we find that 21° 4' 30" lies within the values listed for D. By interpolation we have:-

Minimum Radius R.

Value for 21° 00'	= 1124.64
" " 21° 30'	= 1111.54
" " 21° 4' 30"	= 1122.67 links

Length of transition L.

Value for 21° 00'	= 411.78
" " 21° 30'	= 416.67
" " 21° 4' 30"	= 412.51 links

Tangent length AO.

Value for 21° 00'	= 415.06
" " 21° 30'	= 420.14
" " 21° 4' 30"	= 415.82 links

Deviation angle	= 21° 04' 30"
Speed Value	= 45 m.p.h.
Min. Radius	= 1122.67 lks.
Max. Super.	= 1.09" per foot
Transition length L	= 412.51 lks.
Tangent length	= 415.82 lks.
I.P. to C.P.	= 25.66 lks.

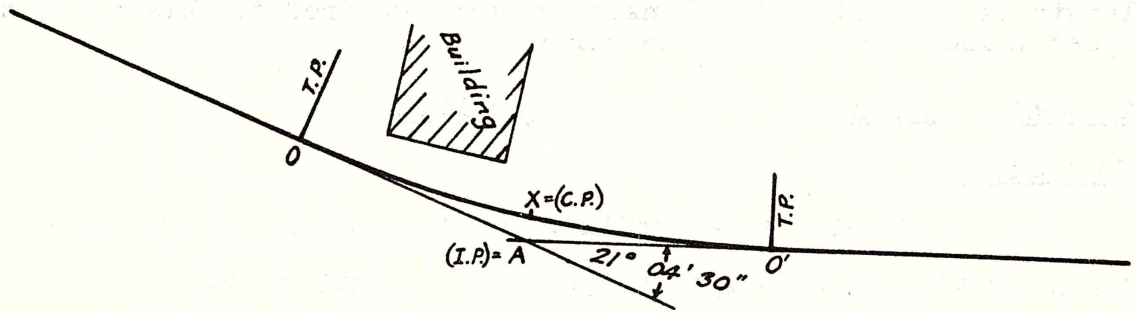


Fig. 9. Example II
Curve Transitional Throughout.

Distance from the Intersection of the straights to the centre point of the Curve (AX).

Value for 21° 00'	= 25.52
" " 21° 30'	= 26.45
" " 21° 4' 30"	= 25.66 links

Total length of Curve.

Total length $= 2L = 2 \cdot (412.51) = \underline{825.02}$

Calculation of the Distance "x"

$$x = 412.51 \cdot \frac{0.375}{1.09} = \underline{142.0 \text{ links}}$$

Superelevation.

The maximum superelevation at the centre point of the curve, obtained from the superelevation and widening diagram = $\underline{1.09'' \text{ per foot}}$

Superelevation at a point 200 lks. from the tangent point $= \frac{1.09''}{1} \cdot \frac{200}{412.51}$

$= \underline{0.528'' \text{ per foot}}$

Widening.

Maximum widening at the centre point of the curve, obtained from the superelevation and widening diagram = 1.5 feet (i.e. 9 ins. on each side)

Widening at a point 200 links from the tangent point $= \frac{18}{1} \cdot \frac{200}{412.51} = 8.73''$

= 8" approx. (i.e. 4" on each side)

Example III.

Curve with Limited Tangent Length.

It is required to insert a Transition Curve between two tangents where the deviation angle is 23° 10' 30", and the tangent length is limited to 460 links; it being desired to obtain the maximum speed value possible for the curve.

Tangent Length AO = 460 links.

Unit Chord

Referring to Appendix II of the tables, we obtain:-

Value of AO when D is 23° 10' 30" = 3.8440 units

3.844 units = 460 links.

and 1 unit = 119.667 links. } = Unit Chord.
= 78.9803 feet. }

Interpolating in Appendix II to obtain values of L, R and AX where D = 23° 10' 30" and multiplying each by the value of the Unit Chord in links (119.667):-

Length L. = 3.8070 units = 455.57 links.

Radius R = 9.4238 units = 1127.72 links.

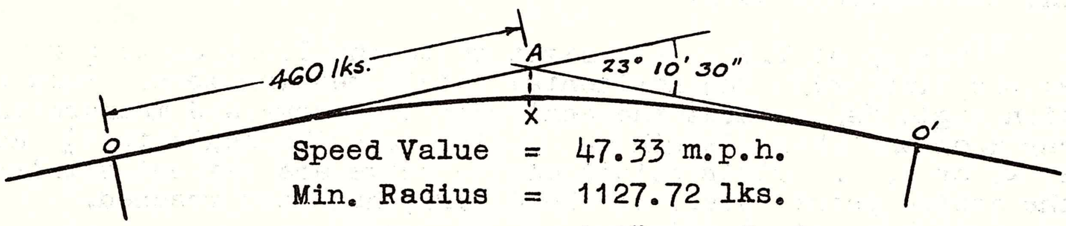
I.P. to C.P. (AX) = 0.2613 units = 31.27 links.

Superelevation and Widening.

Calculated as in the other examples.

Co-ordinates X and Y.

Taken from Appendix I and converted from units to feet or links as required.



Speed Value	=	47.33 m.p.h.
Min. Radius	=	1127.72 lks.
Max. Super.	=	1.2" per foot.
Transition length	=	455.57 lks.
I.P. to C.P.	=	31.27 lks.

Fig.10. Example III

Curve with Limited Tangent Length.

Speed Value.

Referring to Table B we find that the value of "e" for a unit chord of 78.9803 ft. will be 1.5. (If necessary, interpolate for the exact value of e and s^3).

$$17 \; c^2 = s^3 \quad \text{(from appendix I)}$$

$$\text{i.e.} \quad s^3 = 17 \; . \; 78.9803^2$$

$$= 106044.09$$

$$\therefore \quad s = \underline{47.33 \text{ M.P.H.}}$$

---

4. FIELD PROCEDURE FOR SETTING OUT TRANSITION CURVES:

Transition curves can be set out accurately, either by deflection angles and polar rays, or by deflection angles and chords. Setting out transition curves by co-ordinates is not normally advisable, but in special circumstances, and where the deflection angle is small or the y co-ordinates are very short, this method may be used. These methods apply to curves transitional throughout, and also to the transitions at the ends of central circular arcs.

(a) For all Transitions:

(i) Preliminary. The first step is to decide a suitable spacing for the pegs, taking into consideration the type of country. The "setting out" chord length between adjacent pegs shall be; either the unit chord length shown in table B for the particular speed value, or will be a convenient fraction of the unit chord. (e.g., $\frac{\text{unit chord}}{2}$ or $\frac{\text{unit chord}}{4}$)

Reference is then made to the required speed value section of table A. Lengths of curves in terms of the unit chord are on the extreme left. Choosing curve lengths, corresponding to multiples of the "setting out" chord length, note down in the field book; the deflection angles, polar rays, and the co-ordinates if they are required, up to the point of minimum radius.

(ii) Instrument Work. Set up the theodolite at the intersection point (I.P.) or an accessible peg on the traverse line, and measure out the tangent lengths, inserting tacked pegs at the tangent points, $T.P._1$ and $T.P._2$. Where possible, the centre point of the curve should be set out from the I.P., by bisecting the intersection angle and measuring the distance to it.

Set up at $T.P._1$ and sight on to the I.P., or to a convenient peg one line, with the horizontal circle reading zero. Turn off deflection angle "a_1" towards the centre of the curve and measure the polar ray ρ_1 or the chord length c_1. Increase the angle to "a_2" and measure ρ_2 or c_2. Other points on the curve are set out similarly, until the centre point (C.P.) or the C.T.P., has been reached. (Note: $c_1 = c_2 = c_3$ etc.)

The instrument is then set on $T.P._2$ and the other transition set out in a similar manner.

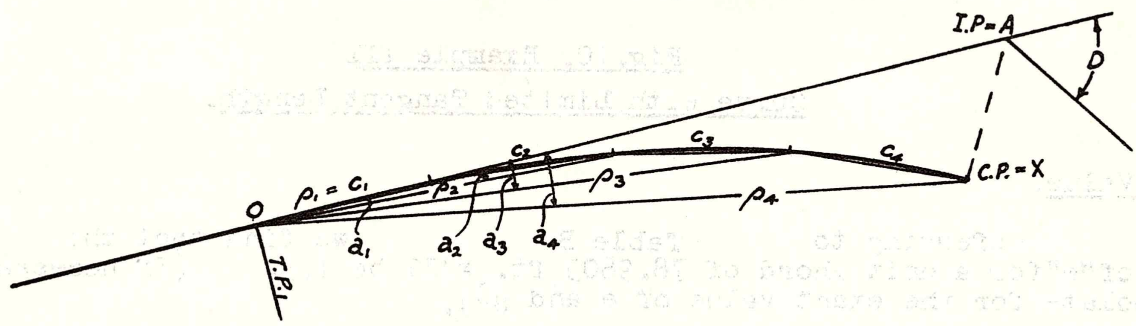


Fig. 11

Setting out a Transition Curve.

If obstacles prevent the complete half of a curve being laid out from the T.P., the instrument will have to be moved to an intermediate peg Q. Set the instrument up at Q, with the horizontal circle reading $(180^\circ + a_Q)$ for a curve to the right, and $(180^\circ - a_Q)$ for a curve to the left, and take a backsight on the T.P. Transit, and turn the telescope towards the main tangent through an angle equal to "a_Q" (the deflection angle to Q from the T.P.). The line of sight will now be parallel to the main tangent and the horizontal circle will read zero. Co-ordinates with Q as origin must be calculated for the centre point or C.T.P., and any intermediate pegs (Q_1, Q_2, etc.). The angles θ_1, θ_2, etc., and distances Z_1, Z_2 etc., must then be computed before Q_1, Q_2 etc. can be set out.

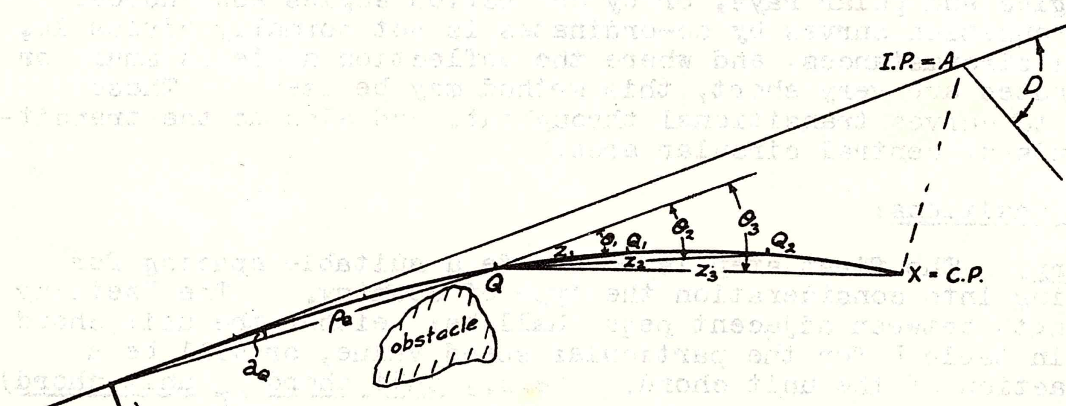


Fig. 12.

Setting out a Transition Curve where Obstacles Prevent the Complete Half being Laid out from the Tangent Point.

(b) <u>For a Curve with Central Circular Arc.</u>

(i) <u>Preliminary</u>. From the plan obtain the radius of the central circular arc and the chainages of the common tangent points, C.T.P.$_1$ and C.T.P.$_2$). Unless the circular arc is very short, it is desirable to set out the curve in even chainages. First determine the distance from C.T.P.$_1$ to the next even chainage, decide on the spacing of the following pegs (e.g., 50 lks. or 100 lks.) and finally determine the distance from the last even chainage to C.T.P.$_2$. These distances are those calculated around the circumference (i.e., arcs of a circle).

Calculate the deflection angle from the C.T.P. to each point on the curve by the following formula:-

$$\delta = 1719 \frac{A}{R} \text{ minutes}$$

Where δ = the deflection angle in minutes, A = the arc length, and R = the radius in the same units as the arc length.

A slide rule is very convenient and reasonably accurate for these calculations, but if the curve is a very long one, greater accuracy can be obtained by the use of logarithms. Check by adding the deflection angles as follows:-

$$\delta_1 + \delta_2 + \delta_3 - - + \delta_x = \frac{D - 6a}{2}$$

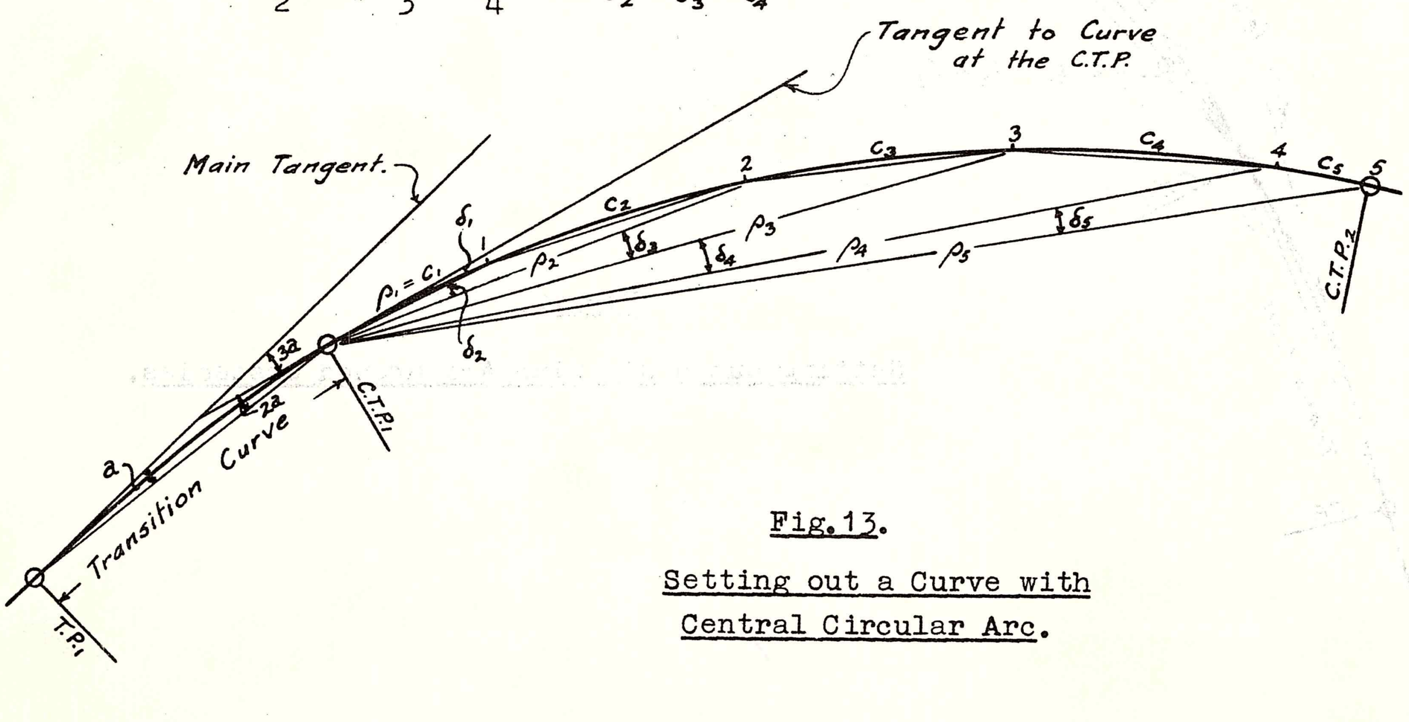
Where D = the total deviation angle, and "a" = the angle from the tangent point to the C.T.P. <u>Note</u>: For circular curves <u>not transitioned</u> at each end, the sum of the deflection angles equals half the deviation angle and the tangent lengths are equal to R tan $\frac{D}{2}$.

Circular arcs can be set out by deflection angles and chords or by deflection angles and rays. Fewer calculations are involved when a curve is set out by chord lengths, but three men are required in the survey party. Provided that the chord lengths are not too long, the chord length may be assumed as equal to the correct arc length, for setting out purposes. Where it is necessary to use the ray method, the lengths of rays can be calculated from the following formula:-

$$\rho = 2R \sin \left(\begin{array}{l}\text{sum deflection angles from the tangent} \\ \text{at the C.T.P. to the required ray.}\end{array}\right)$$

$$\text{e.g. } \rho_4 = 2R \sin (\delta_1 + \delta_2 + \delta_3 + \delta_4)$$

Where $c_2 = c_3 = c_4$:- $\delta_2 = \delta_3 = \delta_4$



<u>Fig. 13.</u>

<u>Setting out a Curve with Central Circular Arc.</u>

(ii) Instrument Work. The two transitions are first set out as described in section (b), and tacked pegs at C.T.P.$_1$ and C.T.P.$_2$ inserted.

With the theodolite stationed at C.T.P.$_1$ and the circle reading $(180^\circ - 2a)$ for a curve to the right, or $(180^\circ + 2a)$ for a curve to the left; take a backsight to T.P.$_1$. Transit, and turn off an angle of 2a towards the centre of the curve. The line of sight will now be along the tangent to the circular arc, and the reading of the horizontal circle will be zero. To locate point 1, turn off an angle δ_1 towards the centre of the curve and measure the chord length c_1 or the ray ρ_1. Increase the angle by δ_2 to $(\delta_1 + \delta_2)$ and measure c_2 or ρ_2 to locate point 2. The other points on the circular arc are set out in a similar manner.

If the full length of the circular arc cannot be set out from C.T.P.$_1$, it will be necessary to set up at C.T.P.$_2$ to complete the curve. If the whole curve cannot be set out from the two C.T.P's, the instrument must be set up at one of the intermediate pegs and the following method adopted. Take a backsight on to the C.T.P., with the horizontal circle set on $(180^\circ - \theta)$ for a curve to the right, or $(180^\circ + \theta)$ for a curve to the left. Transit, and turn off an angle of θ towards the centre of the curve. The line of sight will once more be along the tangent to the circular arc, and the reading on the horizontal circle will be zero. Continue to set out the other points as before, after calculating the deflection angles from the new instrument station.

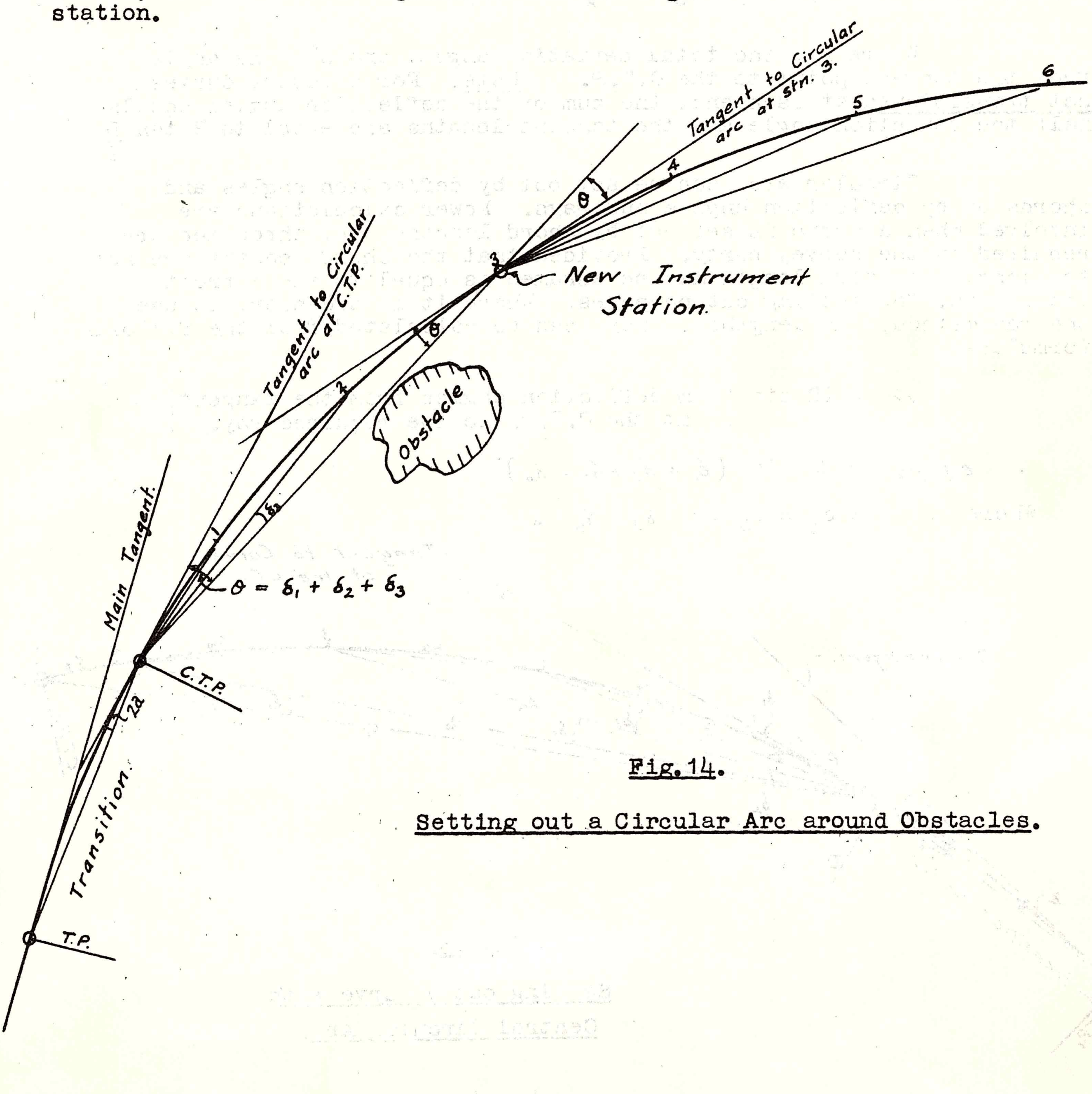


Fig. 14.

Setting out a Circular Arc around Obstacles.

PART II VERTICAL CURVES

1. INTRODUCTION.

Vertical curves shall be inserted at each change of grade. The curve adopted for this purpose is the parabola, the properties of which are shown in Fig. 15.

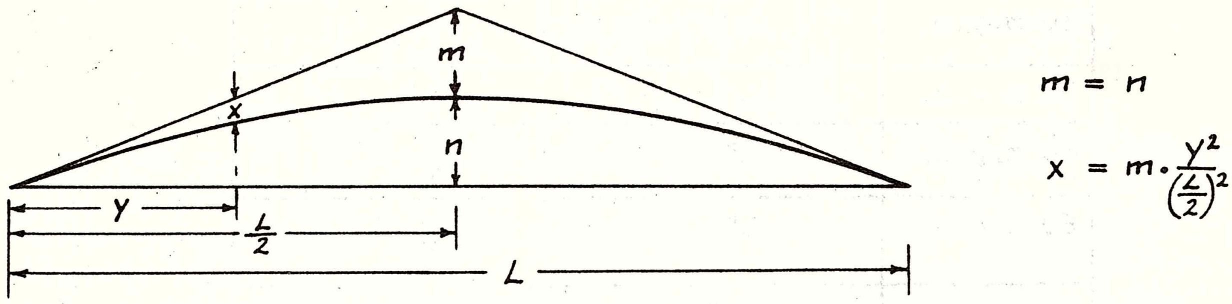


Fig. 15.

Properties of the Parabola.

The curve illustrated is constructed between two equal gradients with the middle ordinate vertical . Most highway vertical curves are sited between unequal gradients and as the ordinates are assumed vertical, there is consequently a slight distortion. However, the errors due to this distortion may be neglected for all practical purposes.

The length of a vertical curve depends on the algebraic difference in grades and the safe stopping sight distance, which increases with the speed value. A diagram showing the minimum lengths of vertical curves for various speed values and algebraic differences in grade is included in these standards. However, with due regard to the cost involved, vertical curves should be designed as long as possible.

Summit Vertical Curves. The lengths of these curves can be obtained directly from the diagram. They are calculated to satisfy safe stopping sight distances; where the sight distance S, is taken as the average distance from a driver's eye at 4'-0" above road level, to an object of height 4" above road level.

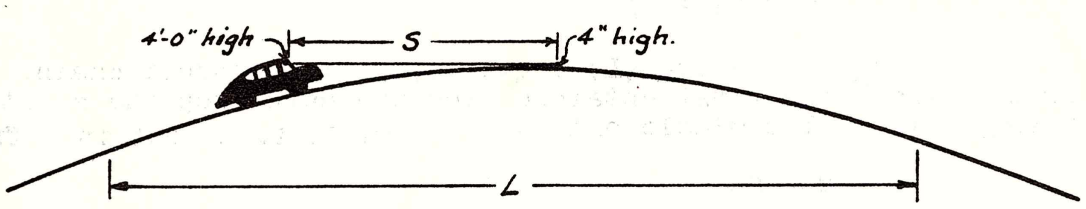


Fig. 16.

Summit Vertical Curve.

Sag Vertical Curves. For uniformity these are usually designed equal in length to the equivalent summit curves, but where necessary, reductions in length can be applied to curves where the speed value is more than 40 m.p.h. The minimum lengths of sag curves are calculated to satisfy safe stopping sight distance at night, comfort and appearance. The following table gives the proportion of minimum length of sag curves to minimum length of summit curves.

Speed Value.	Length of Summit Curve	Length of Sag Curve
30 m.p.h.	L	$\frac{5}{4}$ L
40 m.p.h.	L'	L'
50 m.p.h.	L"	$\frac{7}{8}$ L"
60 m.p.h.	L"	$\frac{2}{3}$ L"'
70 m.p.h.	L""	$\frac{7}{12}$ L""

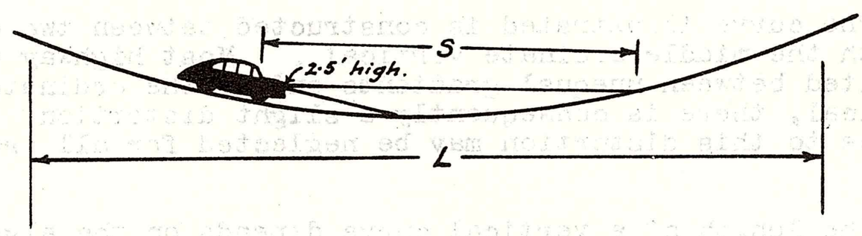


Fig.17.

Sag Vertical Curve.

2. CALCULATIONS.

(a) Longitudinal Measurements in Chains. Where possible, arrange the grades so that they intersect at an even chainage. Then determine the following:-

The algebraic difference in grades (D) in feet per chain. Ascending grades are considered as positive and descending grades as negative. Where G_1 and G_2 are the gradients in feet per chain.

$$D = G_1 \pm G_2$$

The length of curve (L) required, to the nearest chain. (The minimum value being first obtained from the safe stopping sight distance diagram). The middle ordinate (M) in feet, is obtained from the formula:-

$$M = \frac{DL}{8}$$

Other ordinates x_1 and x_2 etc., at distances y_1 and y_2 etc. from the ends of the curve are calculated from the following formula:-

$$x = \frac{D}{2L} \cdot y^2$$ (N.B. Both y and L are in chains; x is in feet)

The levels at each grade point are then determined and the corresponding ordinates added or subtracted to give final formation levels.

(b) Longitudinal Measurements in Feet. Where possible, arrange the grades so that they intersect at an even hundred feet or fifty feet. Then determine the following:-

The algebraic difference in grades (D) in feet per hundred feet (i.e., percentages). Ascending grades are considered as positive and descending grades as negative. Where G_1 and G_2 are the gradients in feet per hundred feet (i.e., percentages) ,

$$D\% = G_1\% \pm G_2\%$$

The length of the curve (L) required, to the nearest fifty feet. (The minimum value being first obtained from the safe stopping sight distance diagram).

The middle ordinate (M) in feet is obtained from the formula:-

$$M = \frac{DL}{800}$$

Other ordinates x_1 and x_2 etc. at distances y_1 and y_2 etc. from the ends of the curve are calculated from the following formula:-

$$x = \frac{D}{200L} . y^2 \quad \text{(N.B. y,x,and L are in feet)}$$

The levels at each grade point are then determined and the corresponding ordinates added or substracted to give the final formation levels.

EXAMPLE.

Assume an ascending grade of 1 in 20 meets a falling grade of 1 in 40. The desired speed value is 50 m.p.h., which requires a minimum stopping sight distance of 350 feet.

1 in 20 = 3.3 feet per chain

1 in 40 = 1.65 feet per chain

Algebraic difference in grades (D) = 3.3 - (-1.65)
= 4.95 feet per chain.

Referring to the safe stopping sight distance diagram, the minimum length of curve is 10.5 chains. Balancing of quantities and other factors are considered and it is decided to use a 12 chain curve.

$$\text{Middle ordinate (m)} = \frac{DL}{8} = \frac{(4.95).12}{8} = 7.43 \text{ feet}$$

$$\text{ordinate } (x_1) = \frac{D}{2L} . y_1^2 = \frac{4.95}{2(12)} . (3.61)^2 = 2.69 \text{ feet}$$

$$\text{ordinate } (x_2) = \frac{D}{2L} . y_2^2 = \frac{4.95}{2(12)} . (2.39)^2 = 1.18 \text{ feet}$$

Other ordinates are calculated similarly.

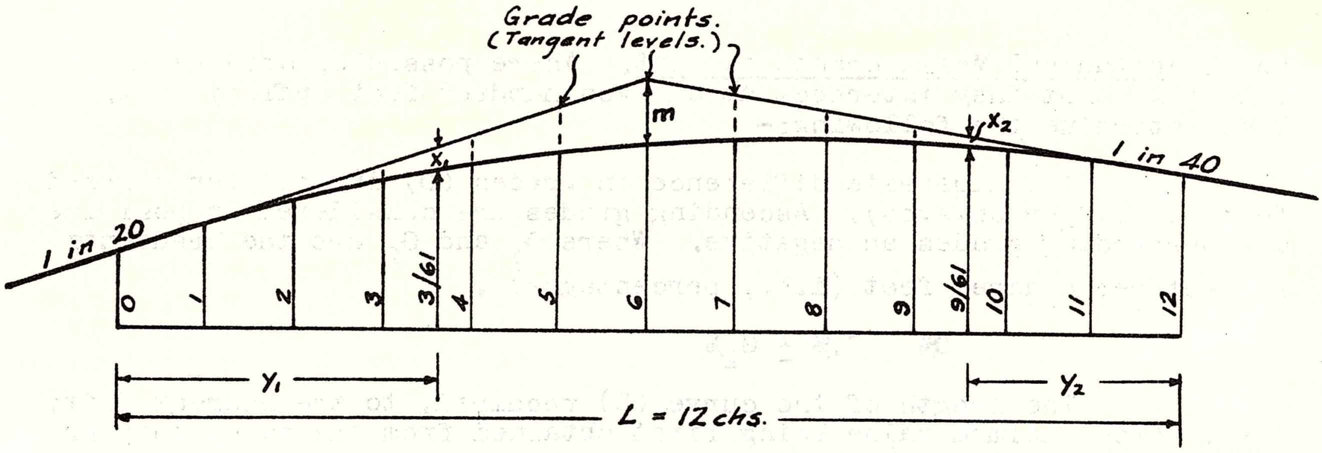


Fig.18

Example - Vertical Curve.

Formation levels can be computed by tabulating as follows:-

Chain Peg	Grade	Tangent Levels	Correction (Ordinate)	Formation Levels
0		100.00	0	100.00
1		103.30	0.21	103.09
2		106.60	0.83	105.77
3		109.90	1.86	108.04
3/61	1 in 20	111.91	2.69	109.22
4		113.20	3.30	109.90
5		116.50	5.16	111.34
6		119.80	7.43	112.37
7		118.15	5.16	112.99
8		116.50	3.30	113.20
9		114.85	1.86	112.99
9/61	1 in 40	113.84	1.18	112.66
10		113.20	0.83	112.37
11		111.55	0.21	111.34
12		109.90	0	109.90

SLIDE RULE METHOD.

Vertical curves can be quickly calculated with the use of a slide rule.

Find the algebraic difference in grades, length of curve and middle ordinate in the ordinary manner.

To calculate the other ordinates:

(a) Set the cursor on scale D at half the curve length.

(b) Set the middle ordinate on scale B at the cursor opposite the half curve length.

(c) Set the cursor on the first distance on scale D.

(d) Read the answer on scale B.

(e) Repeat for all the other distances from each end of the curve, setting cursor on scale D and reading the answer from scale B.

Co-ordination of Horizontal and Vertical Alignment:

All horizontal and vertical curves should comply with their respective standards, but co-ordination of vertical and horizontal alignment is also desirable.

Where horizontal and vertical curves overlap, it is advisable to rearrange them to coincide approximately in regard to both length and position. To comply with this condition, it will often be found necessary to lengthen vertical curves. This arrangement will greatly improve the appearance of a highway. Although it may not always be possible to make horizontal and vertical curves coincide, the placing of a short vertical curve within a long horizontal curve should be avoided. Where horizontal curves coincide with summit vertical curves, the superelevation should be designed to a lower value than would otherwise be employed.

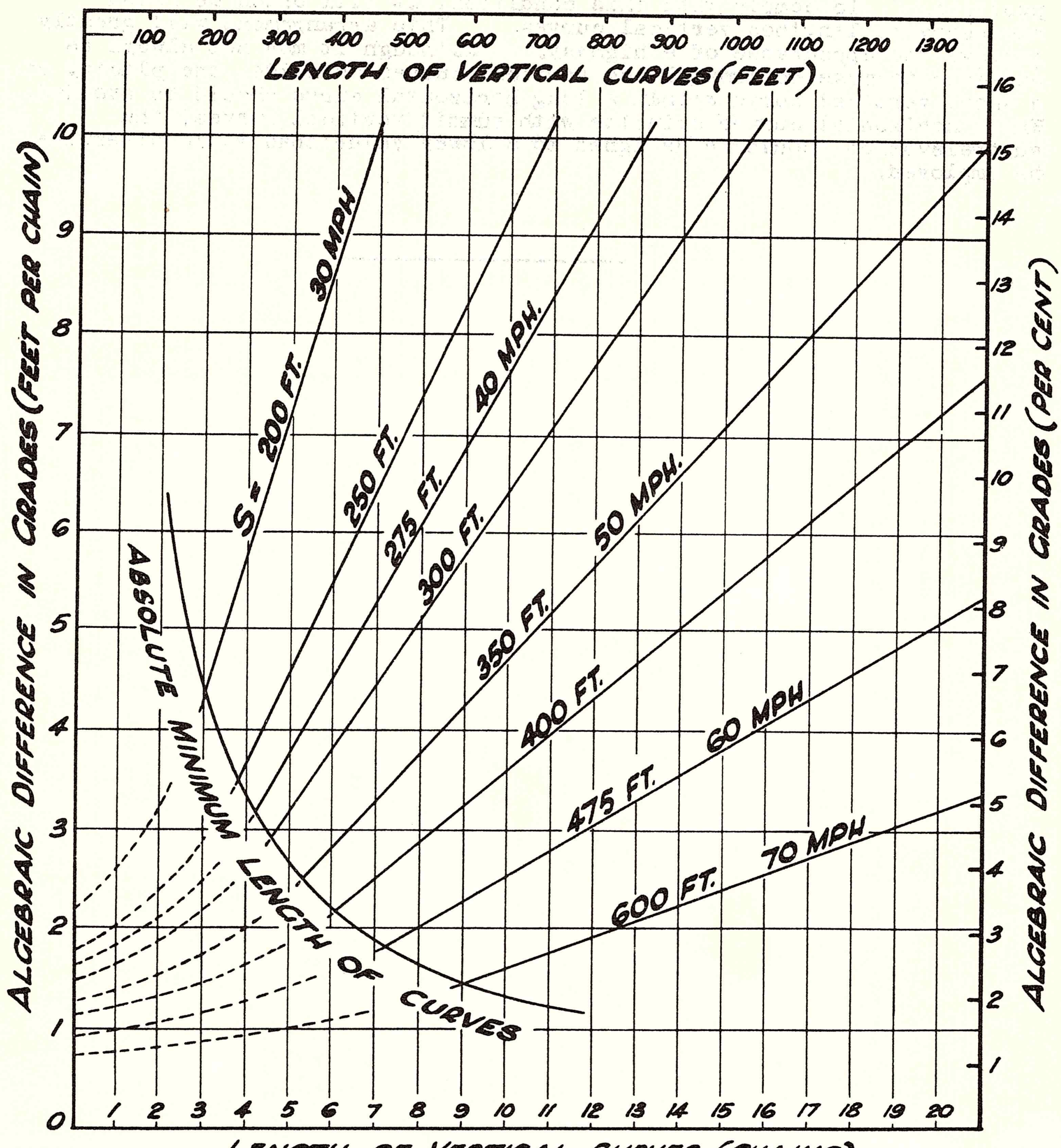
NRB.

April
1955

SAFE STOPPING SIGHT DISTANCES

(Height of eye 4 feet — Height of object 4 inches)

MINIMUM LENGTH OF SUMMIT VERTICAL CURVES



MINIMUM LENGTH OF SAG VERTICAL CURVES

Speed Value	Length of Summit Curve	Length of Sag Curve
30 MPH	L	$\frac{5}{4}L$
40 MPH	L	L
50 MPH	L	$\frac{7}{8}L$
60 MPH	L	$\frac{2}{3}L$
70 MPH	L	$\frac{7}{12}L$

This table gives the proportion of minimum length of sag curves to minimum length of summit curves.

R. E. Owen, Government Printer, Wellington, New Zealand, 1956